Studies in Conflict & Terrorism, 30:635–646, 2007

ISSN: 1057-610X print / 1521-0731 online
DOI: 10.1080/10576100701385974

Routledge
Taylor & Francis Group

The Torturer's Dilemma: A Theoretical Analysis of the Societal Consequences of Torturing Terrorist Suspects

VLADIMIR A. LEFEBVRE

School of Social Sciences
University of California
Irvine, California, USA

JONATHAN DAVID FARLEY

Center for International Security and Cooperation
Stanford University
Stanford, California, USA
and
Department of Mathematics
Harvard University
Cambridge, Massachusetts, USA

Terrorism today is one of the main threats to Western civilization. Almost as dire a threat is the fight against terrorism itself. A broad spectrum of antiterrorism measures may significantly change the nature of society by destroying those forms of cooperative activity that underlie Western culture. This threat becomes obvious with an analysis of mathematical models of human moral behavior.

Consider a moral dilemma formulated by Harvard Law Professor Alan Dershowitz:

> [A] captured terrorist knows the location of a ticking bomb that threatens hundreds of innocent lives; the only way to prevent the mass murder is to torture the terrorist into disclosing the bomb's location; there is no time for reflection; a decision must be made. Does the noble end of saving innocent lives justify the ignoble means of employing torture?[1]

This article, using a mathematical model of the subject, demonstrates the consequences that would follow should such considerations become the cultural norm. Although this

Received 21 July 2006; accepted 29 July 2006.
The authors would like to thank Dr. Victorina Lefebvre, Ms. Jane Ryan, and Dr. Stefan Schmidt. The second author thanks Professor Phil Heymann of Harvard Law School for discussions regarding this article.
Address correspondence to Jonathan D. Farley, Department of Mathematics and Computer Science, The University of the West Indies, Mona, Kingston 7, Jamaica, West Indies. E-mail: lattice.theory@gmail.com

discussion has a moral dimension, this article uses mathematics merely as a way of clearly expressing its assumptions and conclusions. The first part of the article explains the main analytical tools. The second part examines two types of ethical systems delineated by the model. It will show how, with simple assumptions, one can anticipate the macroscopic effects in a society of the decisions made by individuals if that society is governed by one or the other ethical system. Finally, the article looks at some empirical data to support the existence (and positive or negative consequences) of the two ethical systems. Implications for legal practice are then suggested.

Algebraic Formulation of a Moral Calculus

One cannot do experiments on societies. One can, however, do thought experiments. The proposed model is exceedingly crude, but it is hoped that it may at least shed some light on the possible consequences of altering the ethical system that seems to underlie society. In this model, assume that all choices are binary: individuals divide the universe into "good" and "evil"; the model only considers relationships between the subject and one other; relationships can take one of two forms: cooperation (compromise) or conflict. The main tool used is reflexive theory.

Broadly speaking, reflexive theory, developed by Lefebvre in the early 1960s and used by the Soviet military establishment, is a branch of mathematical psychology in which agents do not necessarily act rationally, as is assumed in game theory, but instead act according to their image of the world and their image of their adversary's image of the world.[2] It thus plays a similar role with respect to the discussion of *values* that Boolean algebra has played with respect to the discussion of truth.[3] For the sake of convenience, the authors use algebraic notation to describe terms that are laden with meaning in natural language, such as "good" or "evil," without, however, making any value judgements as to the *actual* goodness or evil of the "ethical" objects considered.

This section begins by clarifying the difference between two important concepts: that of the "value system" and that of the "ethical system." Every society has a list of elements that may be coded, for want of better terms, "good" or "evil." Elements of such a list might include natural disasters (for instance, "earthquake," "famine"), social eruptions (for instance, "military defeat"), sacred objects (e.g., "cathedral"), social goals (e.g., "democracy," "prosperity"), different types of human behavior or characteristics ("deceitful," "murder," "selflessness," "kindness"). The authors define a *value system* to be such a list. It is assumed that a value system is obtained by every member of a given society (usually in the process of reaching maturity and acquiring an education); it becomes, as a rule, an inseparable attribute of an individual's personality.

A value system is rarely sufficient for the resolution of even the most rudimentary of social problems, however. Individuals usually operate not with a single value but with complexes of values that are interrelated.[4] Consequently, a need exists for rules that permit individuals to assign codes of "good" and "evil" to elements by combining the codes from their value system. The authors define the term *ethical system* to mean a set of rules that govern the way values are combined. Two societies with identical families of value systems may differ significantly if their ethical systems differ.

The model under consideration here is based on the assumption that the subject possesses an inborn mechanism capable of "computing" the choice leading to a particular action by exploiting the relations between her value system and her ethical system.[5]

The scheme for such a mental computational process is illustrated in Figure 1. The lowest shape (or "mask" if you will) indicates the subject. The two masks above it indicate

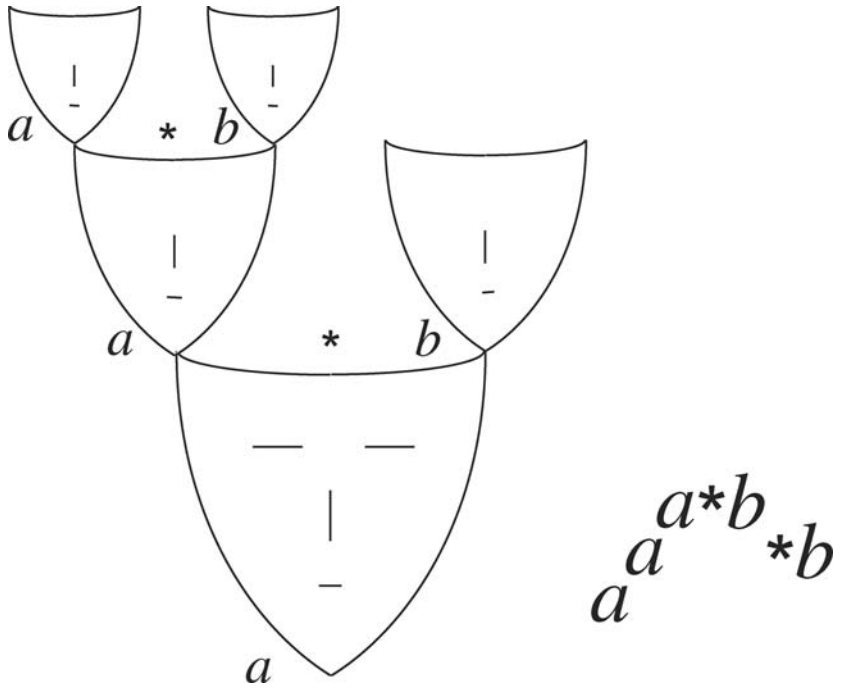

Figure 1. The subject with images of the self and the other. The image of the self, in turn, has its own images of the self and the other. Note that the exponent of a is $(a^{a*b}) *b$.

images: the left one (a) is an image of the self, and the right one (b) is the subject's image of another. The image of the self has its own image of the self (the left mask on the third tier), as well as the image of the other (the right mask on the third tier). The asterisk between masks symbolizes a subjective image of the relationship between the self and the other. Each mask can receive an "impulse" from the outer world. These impulses correspond to Boolean values 0 and 1.[6] For the lower mask, the value of 1 is an impulse that proceeds from the real world and inclines the subject to perform a positive act; the value of 0 is an impulse that inclines the subject to perform a negative act. The values of 0 and 1 on the second tier indicate the subject's *notions* of the impulses proceeding from the outer world toward herself and the other. Finally, the values of the masks on the third tier reflect the notions of the subject concerning impulses from the outer world *of which she is aware*.[7]

Algebraic notation can simplify these discussions. The symbol "$*$" will be used to denote, depending on the circumstance, the symbol "$+$" or "$\bullet$" from Boolean algebra. There is an additional operation of exponentiation that corresponds to the mental phenomenon of self-awareness. This is what prevents the impulses from the outer world from being replicated in the higher tiers: An act of self-awareness can prevent the transformation of an "evil" intention into an action; the subject feels an accompanying prick of guilt. This is called conscience.[8]

The formula to the right of the masks in Figure 1 corresponds to the subject. Tables 1–3 show the algebraic operations connecting the values of the "components" of the subject's mental system into a unified computational process. By assigning values to a and b, and replacing "$*$" by "$+$" or "$\bullet$," one can calculate the value of the function that corresponds to the formula: 1 signifies that the subject chooses "good," 0 that the subject chooses "evil."

Table 1
Truth tables

a	b	$+$	$\bullet$	a^b
1	1	1	1	1
1	0	1	0	1
0	1	1	0	0
0	0	0	0	1

Analysis of Two Ethical Systems[9]

The authors interpret "+" and "•" depending on which of two ethical systems the subject adheres to. In the first ethical system, "•" is interpreted as a joining together or "combination" of the elements (people or values), therefore "$1 \bullet 0 = 0$" (as depicted in line 2 of Table 1) signifies that a combination of "good" and "evil" is negatively evaluated. In the second ethical system, "+" designates combination, and therefore a combination of good and evil is expressed by "$1 + 0 = 1$" and evaluated positively. To use ordinary language, in the first ethical system, for example, noble ends do not justify ignoble means; in the second, they do.

In the first ethical system, "+" is interpreted as separation, that is, "$1 + 0 = 1$" signifies that the separation of good from evil is evaluated positively (it can only be good to reject the employment of ignoble means to achieve noble ends[10]). In the second ethical system, separation is designated by "•," and "$1 \bullet 0 = 0$" signifies that the separation of good from evil is evaluated negatively (it is bad to restrain from a movement toward noble ends because of the necessity of employing ignoble means).

Let the subject have to choose the relationships "+" or "•" between herself and the other subject, with impulses 0 or 1 coming from the outer world at random. A fragment of Figure 1, shown in Figure 2, represents the subject's image of the self with its own images of the self and the other. The formula to the right of the masks corresponds to this structure. Its value is to be interpreted as the subject's own self-evaluation. For example, when $a = 0$ and $b = 1$,

$$0^{0+1} = 0 \textit{ and } 0^{0 \cdot 1} = 1.$$

See Table 2.

Table 2
The second tier of reflection

a	b	a^{a+b}	$a^{a \bullet b}$
1	1	1	1
1	0	1	1
0	1	0	1
0	0	1	1

Table 3
The third tier of reflection

a	b	$a^{a^{a+b}+b}$	$a^{a^{a \cdot b} \bullet b}$
1	1	1	1
1	0	1	1
0	1	0	0
0	0	0	1

Note that the exponent of a in the third column is $(a^{a+b}) + b$. The exponent of a in the fourth column is $(a^{a \bullet b}) \bullet b$

To summarize: The subject adheres to one of two ethical systems. In each ethical system, there are two ways of combining values from which she can choose. Adhering to an ethical system does not mean that an individual *will* behave in a certain way. She will always be able to choose between "+" and "•." The ethical system merely shows how her choice of action is to be interpreted.

One can see that if the subject chooses "+," she will have a positive self-evaluation three times out of four, but if she selects "•," her self-evaluation will be positive four times out of four, that is, *always*. Therefore, in seeking to have a better view of herself, the subject will prefer "•" to "+" *independently of the ethical system to which she belongs*.

Such a preference has quite dramatic consequences. In the framework of the first ethical system, in order to look better in her own eyes, the subject will aim to develop cooperative relationships with the other subject; in the framework of the second ethical system, she will choose estrangement and conflict.[11]

The authors now conclude that, within the framework of the first ethical system, the subject makes a negative evaluation of using ignoble means in order to achieve noble ends, while inclining toward forming cooperative relationships with the other. In the framework

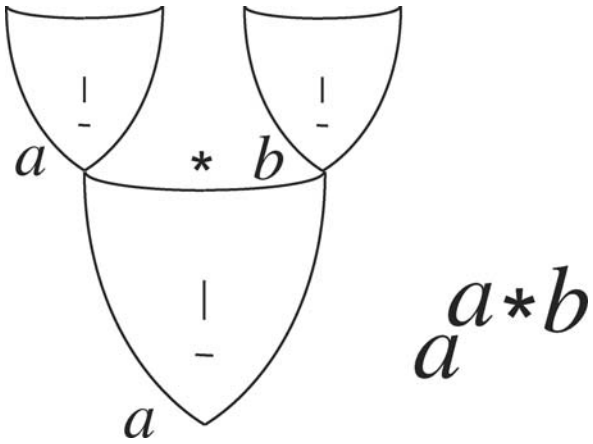

Figure 2. The image of the self with images of the self and the other. When the subject chooses "•" instead of "+," her self-evaluation takes on the value of 1 more often. (See Table 2.) That is, she looks better in her own eyes.

of the second ethical system, the subject makes a positive evaluation of the employment of ignoble means to achieve noble ends, and tends to choose relationships of estrangement from the other.

How will the subject appear in the eyes of others? Table 3 seeks to answer this question. By selecting "+," the subject will perform positive actions only in half the number of cases, whereas by choosing "•," the subject will perform positive actions in three-fourths of the cases. This is true in both ethical systems. Therefore, in the first ethical system, the subject advances in both her own and the other's eyes when she selects the relationship of "union," whereas, in the second ethical system, she advances when she selects "confrontation."

Empirical Observations

Up until now this article has viewed the two ethical systems without prejudice in favor of either one. The article now asks the question of whether one is in a certain sense preferable to the other. It does so first by conducting a thought experiment and then by looking at empirical data, stemming from the Cold War and then from the War on Terror.

Without a priori considerations of utilitarian factors, the possibility is suggested that one ethical system could be better for a society than the other. An example that illustrates this is the following: Consider two roads, with a stream of cars moving along each one. It is natural to suppose that each driver wishes to reach his destination as quickly as possible. It is also natural to suppose that drivers will on occasion impede each other and so, because of that possibility, they will exist in a state of constant rivalry.[12] The drivers on the first road follow the first ethical system: They rise in their own and in other drivers' estimation if they yield. Drivers on the second road follow the other ethical system: They lose face when they yield. It is clear that traffic will move faster on the first road than on the second. Thus, by incorporating what might be called "the desire to preserve personal dignity" into the model, one can understand macroscopic effects. The attempt of individual drivers to save face by rejecting cooperation influences the macroscopic scene; this factor, however, is not considered in traditional economic analysis.[13]

Now one asks if there is evidence for the two ethical systems operating in any human society. The data come from surveys conducted in the late 1970s of a group of recent Soviet émigrés and a group of American citizens. The first series of questions dealt with the combination (or separation) of "good" and "evil." Respondents had to express agreement or disagreement with the use of ignoble means to attain noble ends. The second series of questions involved a moral dilemma. Respondents had to choose between cooperation and conflict in the resolution of the dilemma.

The results are displayed in Tables 4 and Table 5. Overall, the Americans viewed the use of ignoble means to attain noble ends as being negative, whereas the Soviets viewed it as being positive. Moreover, the Americans preferred to resolve the dilemmas by means of compromise, whereas the Soviets preferred conflict. This suggests that the first ethical system dominated in the United States at the time, whereas the second ethical system dominated in the Soviet Union at the time.

Now the discussion returns to where it began, the problem of terrorism. In Saudi Arabia, the second ethical system shapes the government's response to terrorism. This has led to an even more severe response from the terrorists, as the following report from Jamal Khashoggi, media advisor to the Saudi ambassador in Great Britain, reveals:

> By this logic, Saudi citizens everywhere become targets. Blowing up the Saudi passport department could be justified because it provided logistical support in

Table 4
Data from a study comparing American and Soviet attitudes toward ethical compromise and ethical confrontation

	Statements	Americans		Soviets	
		% in agreement	standard error	% in agreement	standard error
1	a. A doctor should conceal from a patient that he has cancer, in order to diminish his suffering.	8.0	±6.8	89.0	±6.7
	b. A doctor should not conceal from a patient that he has cancer, in order to diminish his suffering.	80.5	±9.9	15.8	±8.0
2	a. A malefactor can be punished more severely than the law requires	11.5	±8.0	84.5	±7.8
	b. A malefactor cannot be punished more severely than the law requires	83.6	±9.4	28.0	±9.9
3	a. One may give false evidence in order to help an innocent person avoid jail.	19.9	±8.4	65.0	±10.0
	b. One must not give false evidence even in order to help an innocent person avoid jail.	82.25	±9.6	42.5	±10.7
4	a. One may send a cheat sheet during a competitive examination to a close friend.	8.0	±6.8	62.0	±10.1
	b. One must not send a cheat sheet during a competitive examination to a close friend.	90.3	±7.4	37.5	±10.3

Lefebvre, *Algebra of Conscience*, p. 42 (see note 8). Totals do not necessarily add to 100% because the questions were given to the respondents separately. Therefore, respondents were free to accept, or reject, both options.

> the fight against moujahedeen. A strike against a newspaper or television station could be justified with assertions that it was an instrument of anti-moujahedeen propaganda.[14]

Societies dominated by the first ethical system regulate behavior by the absolute prohibition of certain actions. Societies dominated by the second ethical system seem to be marked by the absence of procedures for resolving conflict in ways that would preserve the dignity of the opponents.[15]

Table 5
Comparison of American and Soviet preferences in choosing a positive individual

	Americans		Soviets	
Statements	% favoring choice (a)	standard error	% favoring choice (a)	standard error
5. A good person in a situation of conflict with an insolent person (a) would not seek compromise with him (b) would seek compromise with him	24.1	±9.05	70.0	±7.48
6. Two terrorists are hijacking a small plane. There is a possibility of killing them without injury to the passengers. Another possibility is to start negotiations first and try to persuade them to surrender. The head of the rescue group made the decision not to negotiate with the criminals. Did he act correctly? (a) yes (b) no	24.7	±8.95	58.5	±8.06

Lefebvre, *Algebra of Conscience*, p. 102 (see note 8).

Implications for Policy and the Law

In summary, this article has introduced a novel method for analyzing ethical dilemmas. Borrowing tools from Boolean algebra, the mathematical formulation of the principles of logic and deductive argument, the authors have crafted a method of reasoning about moral behavior. The basic axioms show that there are two ethical systems and, for each ethical system, two Boolean operations, in which the combination or separation of good and evil is regarded as good, or evil, accordingly. One cannot conduct experiments with societies, nor can one directly observe the "moral deductions" individuals make when deciding what actions to take. This model gives one some ability to peer into this inner realm of conscience. It would be of interest to conduct more surveys like those done of Soviet émigrés in the 1970s, in order to create an "ethical map" of modern-day societies, both in the West and in the Muslim world.

One important consequence of this model is that it enables one to attempt to resolve the situation described by Dershowitz.[16] The second ethical system dominates in a society where torture is permissible, that is, where noble ends justify ignoble means. This model shows that, even when individuals have free choice in such a system (for instance, to torture or not to torture), they will tend toward the negative forms of behavior. In other words,

a declaration that permits the use of "evil" means for "good" ends seems inevitably to *promote* "evil" acts.[17] A court decision establishing a precedent that an individual may torture a terrorist would represent such a declaration.

Mathematics is not morality. But this analysis suggests that changing the law might change a society—from 1, to 0.

Notes

1. Alan M. Dershowitz, *Shouting Fire: Civil Liberties in a Turbulent Age* (Boston: Little, Brown and Company, 2002), p. 470. Drozdova and Samoilov also advocate torturing terror suspects in "Security and Liberty," *Hoover Digest* (1)(2002), pp. 80–90.

2. "The Soviet Union had a propaganda machine second to none. One of its most intriguing methods for managing information and getting people (or an opponent) to perform a certain action was described by the theory of reflexive control (RC). In a military context, it can be viewed as a means for providing one military commander with the ability to indirectly maintain control over his opposing commander's decision process. Reflexive control involves creating a pattern or providing partial information that causes an enemy to react in a predetermined fashion without realizing that he is being manipulated. Its aim above all else is to influence command and control systems and decision makers." Lieutenant Colonel Timothy L. Thomas, "Human Network Attacks," *Military Review* (September–October 1999), p. 31. See also Timothy L. Thomas, "Russia's Reflexive Control Theory and the Military," *Journal of Slavic Military Studies* 17 (2) (April–June 2004), pp. 237–256 and Colonel S. Leonenko, "O Refleksivnoe upravlenie protivnikom (On Reflexive Control of the Enemy)," *Armeyskiy sbornik* (*Army Journal*), No. 8 (1995), quoted in Thomas.

3. Boolean algebra allows for the analysis of the laws of thought by means of algebraic equations without the obfuscation of natural language, with 1 denoting truth and 0 denoting falsehood. In general, no claim is made as to the *actual* truth or falsity of the (usually abstract) sentences under consideration. For instance, the syllogism, "Hannibal is a man; all men are mortal; therefore Hannibal is mortal," has the same *logical* form as the sentence, "Cleopatra is a man; all men are mortal; therefore Cleopatra is mortal." For a primer on Boolean algebra, see Appendix 1.

4. *Value* here means a coding of a particular element of the list as 1 or 0. (This is not to be confused with the using the term *value* to describe the *numbers* 1 and 0.) This article is asserting the obvious fact that for the resolution of most issues it is not enough to know (for instance in the case of the penal system) that it is *wrong* to deny someone his freedom. Trivial examples of dilemmas not soluble with one value alone: the decision to drop (or build) the atomic bomb; a doctor's decision to lie to a terminally ill patient; Huck Finn's decision to aid Jim, and so on.

5. The authors do not posit that an individual is conscious of the actual process by which she reaches decisions, any more than an individual is conscious of the process by which she makes logical deductions. In particular, while the rules of Boolean algebra govern "the laws of thought," it is unlikely that any individual creates truth tables before reaching a logical conclusion.

6. Again, in deductive logic, notions such as truth and falsehood are dealt with simply as constants 1 and 0. This does not mean that the natural language notions of truth and falsehood have no other content. Similarly, this article deals with ethical notions in terms of these constants without disregarding the actual significance of the real-world notions.

7. One could ask why this article does not consider higher orders of reflection. The third tier represents the lowest level at which the subject is aware of the influence of the outer world. Thus, the simplest model capturing the act of self-awareness must rise at least to the third tier. It seems unreasonable to suppose much higher orders of awareness. An analogy is with language: Although English theoretically allows infinitely many levels of recursion ("I think that I think that I think that snow is white"), in practice only two or three levels of nested clauses are used or are even intelligible.

8. V. A. Lefebvre, *Algebra of Conscience* (Dordrecht, Holland: Kluwer Academic Publishers, 2001). For experimental evidence confirming the basic predictions of reflexive theory, see Jack R. Adams-Webber, "Comment on Lefebvre's Model from the Perspective of Personal Construct Theory,"

Journal of Social and Biological Structures: Studies in Human Social Biology 10 (April 1987), pp. 179–189.

9. See Appendix 2 for an explanation of how the two ethical systems and the two Boolean operations arise.

10. This is the position, for instance, of individuals like Dr. Martin Luther King, Jr.

11. All this occurs without any supposition of utilitarian factors.

12. The "war of all against all."

13. Although see Thomas Schelling, *Micromotives and Macrobehavior* (New York: W. W. Norton and Company, 1978). One could of course create a single utility function that attempts to value the desire to preserve personal dignity along with other more straightforward desires, such as monetary gain. Given that utility functions are normally ordinally valued (for instance, real-valued), this seems an awkward and artificial way to combine what are clearly different sorts of values. Multiple scales should be used.

14. Jamal Khashoggi, "A Bleeding Saudi Arabia Is Asking: Just What Do the Terrorists Want?" *Los Angeles Times*, 3 June 2004, p. B13.

15. A conflict is resolved only when one side achieves victory or when the conflict is halted by the authority. If no such authority exists, even minor disagreements could cascade into catastrophes.

16. It is a given that the situation described is a moral dilemma, that each case is unique, and that therefore one must be wary of making conclusions based on abstract theory.

17. One might go further and conclude that the second ethical system nourishes terrorism.

18. An analogy from physics is the following: There are two sorts of physical forces, attraction (combination), as with gravity, and repulsion (separation), as with electromagnetism, when two bodies have like charges.

19. Lefebvre, *Algebra of Conscience*, Chapter II and Conclusion of Part One.

Appendix 1. Primer on Boolean Algebra

Boolean algebra strips language down to its bare logical essentials. Sentences such as "Snow is white" and "John F. Kennedy is alive" have radically different meanings, but, in Boolean algebra, one is only concerned with whether a sentence is true or false. Sentences that are true are said to have the truth-value 1; false sentences have truth-value 0. Such sentences are also called *propositions*, and are given names such as p, q, and so on.

In ordinary arithmetic, there are three major operations for combining numbers: *addition*, *multiplication*, and *exponentiation*. In Boolean algebra, there are three major operations for combining propositions: *disjunction*, *conjunction*, and *implication*. For the first two the same symbols "$+$" and "$\bullet$" are used as in ordinary arithmetic, but the symbols have a very different meaning and should not be confused with the addition and multiplication of ordinary numbers.

Disjunction corresponds to the English *or*. If p is the proposition, "Snow is white," and q is the proposition, "John F. Kennedy is alive," then the statement, "Snow is white or John F. Kennedy is alive"—that is, "p or q"—can be written $p + q$ and is called the *disjunction* of p and q. In this case, $p = 1$ and $q = 0$, and clearly $p + q = 1$ (that is, the statement "Snow is white or John F. Kennedy is alive" is true). In general, the logical proposition $a + b$ is true if at least one of the propositions a and b is true, which fact is represented by the first three columns of the "truth table" of Table 1. There are four possibilities, given two propositions a and b, represented by the four rows of the table: either a and b are both true, or a is true and b is false, or a is false and b is true, or a and b are both false. Note that the logical disjunction does not exactly correspond to all uses of the English word *or*. In ordinary language, *or* sometimes means "one or the other *but not both*." (For instance, "You can have chicken or fish for dinner.") In Boolean algebra, disjunction means "one

or the other or possibly both." This leads to the equation $1 + 1 = 1$, which, of course, is nonsensical for ordinary numbers.

The second major operation, conjunction, corresponds to the English *and*. So the statement $p \bullet q$, read "p and q," means, "Snow is white and John F. Kennedy is alive." In this case, $p = 1$ and $q = 0$ and, clearly, $p \bullet q = 0$. In general, the logical proposition $a \bullet b$ is true if and only if both of the propositions a and b are true. This fact is represented by the first, second, and fourth columns of Table 1. Note that logical conjunction does not exactly correspond to all uses of the English word *and*. For instance, "John baked a cake and Mary ate it" makes perfect sense, whereas "Mary ate a cake and John baked it" has a different meaning, or perhaps none at all.

The third major operation, implication, corresponds to the English construction *if. . .then*. The statement a^b should be read "if b then a," or, in longhand, "if b is true then a must be true." Logical implication corresponds even less than *or* and *and* to its ordinary language counterpart. For instance, it makes sense to state, "If the O ring fails then the space shuttle will explode," but little sense to state, "If the O ring fails then America is over 200 years old." It makes even less sense to state, "If John F. Kennedy is alive, then snow is white." Nonetheless, logicians have decided on the convention that the only case where one can agree that the statement "if b then a" (or "b implies a") should definitely be false is if b is true but a turns out to be false. For the sake of simplicity, in all other cases, as odd as it may sound in English, one says that the statement a^b is true. This is the last column of Table 1.

Despite the unusual definition of implication, it at least has the consequence that the statement "if p implies q, and if q implies r, then p implies r" is always true, no matter what truth-values p, q, and r may have.

Appendix 2. Explication of How the Boolean Operations Arise in Ethical Arithmetic

In the section "Analysis of Two Ethical Systems," it was asserted without proof that there were two ethical systems, and, for each ethical system, two ways of combining values, corresponding to the Boolean operations "$+$" and "$\bullet$." This Appendix explains intuitively why this is so and points toward a mathematical derivation of this fact. It also discusses the operator of awareness, which is denoted by exponentiation, and explains why the authors have defined it as in Table 1.

The authors operate under the assumption that all ethical "objects" can take one of two values, 1 and 0, corresponding to "good" and "evil." When considering two ethical objects at once, however—say, a and b—there needs to be some way of evaluating the entire situation, which is a composite of a and b. This composite is denoted $a*b$.

It seems reasonable to suppose that the composite of two "good" ethical objects or situations will be "good," and the composite of two "evil" ethical objects will be evil. (This is merely saying that good cannot come from evil, or vice versa.) Algebraically, this would be written as $1 * 1 = 1$ and $0 * 0 = 0$. Thus it only remains to consider the value of $1*0$ (which should be the same as $0*1$). This can only have one of two values, 0 or 1; in the former case one gets the operation "$\bullet$," and in the latter "$+$." (See Table 1.) These two operations correspond to two possible ways of combining values that an individual may be using at any given time.

The two *ethical* systems come about when one considers the nature of the composition. The composition operator "$*$" can be considered either "combination" or

"separation"—given two objects that one is considering together, these are the only ways one can think of relating the two, in the simplest possible model.[18] Therefore in one ethical system, "•" stands for combination (and therefore the only other operation, "+," must stand for separation), and in the other ethical system, "+" stands for combination (and therefore "•" stands for separation).

Now one comes to the operator of awareness. In the expression a^b, a corresponds to the subject and b corresponds to the subject's image of herself. Table 1 states that $1^1 = 1$. This is interpreted as follows: If the subject is good and views herself as good, good (for instance, a good action) will result. It also states that $1^0 = 1$: if the subject is good and views her actions as evil, she will correct them and good will result. Similarly, $0^1 = 0$ means that if the subject is evil but views herself as good, then she will not correct her course and evil will result. Finally, $0^0 = 1$ means that an evil subject who disapproves of her own actions will change her course (and the opposite of evil is good).

In actuality, there is a more rigorous and mathematically sound way of proving that, in the authors ethical calculations, the composition operation "$*$" can only be the Boolean operation "+" or "•," and that the operator of awareness must also conform to Table 1. This follows from three axioms. The first is that an individual with a correct image of herself (i.e., who correctly perceives herself to be good if she is good, and to be evil if she is evil), has a higher ethical status than one who does not. By "higher ethical status," the authors mean that the evaluation of a^a is more likely to be 1 than the evaluation of a^{-a}, where "$-a$" is 1 if a is 0 and 0 if a is 1. This corresponds to the fact that an evil individual who sees his actions as evil, like a St. Augustine, is usually then viewed in a better light by the rest of society. The second axiom is that an individual who *thinks* she has an incorrect view of herself, but does not, has a higher ethical status than an individual who has an incorrect view of herself but does not know it. Individuals who are right but still question themselves are usually viewed more highly than individuals who are certain but wrong. Here we are comparing a^x where x is a^{-a}, and a^y where y is $(-a)^{-a}$. These two axioms alone show that the operator of awareness can *only* be the operation given in Table 1.[19]

The third axiom isolates the composition operation "$*$." Suppose a corresponds to the subject and b to the other. An individual with a correct view of herself should have a higher ethical status than an individual with an incorrect view of herself, who is in all other respects the same. Here the authors are comparing a^{a*b} and $a^{(-a)*b}$. One can prove mathematically that only two operations will satisfy this axiom, namely the Boolean operators "+" and "•."